







LIMITING BANGE FOR MEDIUM-RANGE TARGET MODEL

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by R. L. Mitchell MRI Report 149-15 21 March 1978

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Introduction

of N point scatterers, where each scatterer can have an RCS that is aspect dependent. The medium-range constraint assumes that all scatterers on the target are in the linear region of the monopulse receive beam, and all scatterers are illuminated with a constant gain by the transmit beam. The purpose of this constraint is to remove the sensor pointing angles from the real-time computation. In other words, the signal that is generated on the RFSS array is independent of the sensor pointing angles. At shorter ranges where the transmit beam is no longer uniform across the target, or where the monopulse difference beam is not linear, the pointing angles of the sensor beam must be known so that the variable weighting can be implemented in the real-time simulation; moreover, the signals that would be received on each monopulse channel must be separately simulated and radiated into specific points on the receive beam so that each channel receives the proper signal and rejects the others.

The purpose of this memo is to determine the minimum range for the applicability of the medium-range model. A simple Monte-Carlo simulation will be used to accomplish this.

The Target Model

A statistical type target model is assumed. Two scatterers are separated by an angle $\theta_{\rm T}$, in between N-2 scatterers are placed at random. Thus the

^{[1] &}quot;Design Requirements for Simulating Realistic RF Environment Signals on the RFSS," MRI Report 132-44, by R. L. Mitchell and I. P. Bottlik, dated 23 September 1977.

target consists of N scatterers that cover an angular width of $\theta_{\rm T}$. All scatterers are assumed to be of equal RCS on the average, and each is fluctuated with a Rayleigh amplitude and random phase.

The Antenna Patterns

The sum channel two-way voltage antenna pattern is assumed to be

$$G_{\Sigma}(\theta) = 1 - \beta \theta^2 \tag{1}$$

and the two-way difference pattern

$$G_{\Lambda}(\theta) = k(\theta - \alpha \theta^3)$$
 (2)

Therefore, if the complex voltage is V_k on the kth scatterer at an angle θ_k , then the received voltages on the two channels are

$$V_{\Sigma} = \sum_{k} (1 - \beta \theta_{k}^{2}) V_{k}$$
 (3)

$$v_{\Delta} = k \sum_{k} (\theta_{k}^{-\alpha} \theta_{k}^{3}) v_{k}$$
 (4)

For the purpose of this investigation we have assumed

$$\alpha = 1.70/6_{3dB}^2$$
 (5)

$$\beta = 1.37/\theta_{3dB}^2$$
 (6)

where θ_{3dB} is the one-way half-power beamwidth. These values are typical of many tracking radars. The constant k will factor out of the problem later.

The Estimate of Angle

We assume that the boresite of the antenna is pointing exactly at the center of the target (midway between the end points). The estimate of angle is assumed to be

$$\hat{\theta}_{ACT} = \frac{1}{k} \operatorname{Re} \{ V_{\Delta} / V_{\Sigma} \} \tag{7}$$

where the subscript ACT denotes the actual (assumed) target, in contrast to an approximate one based on the medium range model.

The Medium-Range Model

A glint centroid will be calculated for the target that is based on $G_{\Gamma}(\theta)=1$ and $G_{\Lambda}(\theta)=\theta$. Thus the compostive signal

$$v' - \sum_{k} v_{k}$$
 (8)

will be radiated from the angle

$$\theta' = \operatorname{Re} \left\{ \frac{1}{v}, \sum_{k} \theta_{k} v_{k} \right\} \tag{9}$$

Now if we use the antenna patterns in (1) and (2), and the formula for the estimate of the angle, we have

$$\hat{\theta}_{APP} = \theta' \frac{1-\alpha(\theta')^2}{1-\beta(\theta')^2} \tag{10}$$

Thus we will compare $\hat{\theta}_{APP}$ with $\hat{\theta}_{ACT}$ to determine where the medium range model breaks down.

Results

In Tables 1 through 6 we show the results of 20 statistical replications of a target consisting of N=5 scatterers, where the target width varies from

 $\theta_{\rm T}/\theta_{\rm 3dB}$ = .25 to 1.50 (the glint angles $\theta_{\rm ACT}$ and $\theta_{\rm APP}$ are designated as ACTUAL and APPROX, each being normalized to the half-power width). For a target width of 25% of the beamwidth (Table 1) the peak error is .003 (or .3% of the beamwidth), which is negligible. For a target width of 50% of the beamwidth (Table 2) the peak error is over 100% of the beamwidth (REP 20); however, the actual glint for this case is also large, amounting to 66% of the beamwidth. In practice, we can tolerate a large error if the glint angle is also large. It is more important to keep the errors small when the glint angles are small. Thus REP 12 in Table 2 represents probably the most severe error, which is 2.8% of the beamwidth when the actual glint is 14% of the beamwidth.

If we rule out those replications where the actual glint is larger than half of the target width, we can construct the following table

Target Width	Peak Error
.25	.002
.50	.028
.75	.076
1.00	.184

All of these errors are negligible. However, when we go to a target of width 1.250 $_{3dB}$ (Table 5) we observe several large errors, even when the actual glint is small. For example, on REP 16 the actual glint is only 7.7% of the beamwidth, but the error is over 2 beamwidths. Clearly, the model breaks down for $\theta_T = 1.25\theta_{3dB}$. The actual point at which the model breaks down lies somewhere between $\theta_T = 1.00\theta_{3dB}$ and $\theta_T = 1.25\theta_{3dB}$. Variations in the antenna patterns and formulas for measuring angle will impact on a precise determination of where the model breaks down, but we can state conservatively that the model is valid as long as $\theta_T \leq \theta_{3dB}$.

In order to test the effect of the number of scatterers in the model, we repeated the previous simulation for N=10. The results are shown in Tables 7 through 12. No major discrepancies are noted from the previous conclusions.

Table 1. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}$ = 0.25 (N=5, all angles normalized to $\theta_{\rm 3dB}$)

REP	ACTUAL	APPROX	DIFF		. SCATTE	RER LOC	ATION	
1	. 051	. 051	000	-, 125	. 020	. 072	. 113	. 125
2	- . 057	058	- . 001	125	123	 051	012	. 125
3	. 043	. 043	. 000	-, 125	056	049	. 047	. 125
4	- . 074	073	. 001	125	092	029	. 083	. 125
5	- . 175	178	003	125	 100	- . 056	. 021	. 125
6	. 167	. 165	- . 001	125	- . 104	. 030	. 123	. 125
7	. 168	. 169	. 001	125	. 048	. 109	. 120	. 125
8	. 002	. 002	000	125	092	072	. 091	. 125
• 9	. 015	. 015	. 000	125	120	. 010	. 080	. 125
10	01B	019	000	125	046	. 066	. 110	. 125
11	. 041	. 041	. 000	125	- . 053	. 021	. 101	. 125
12	. 095	. 096	. 001	125	026	001	. 097	. 125
13	045	047	002	125	. 024	. 093	. 108	. 125
14	. 085	. 035	. 000	- . 125	073	009	. 097	. 125
15	069	071	002	- . 125	013	 011	. 032	. 125
16	03 5	034	. 001	- . 125	066	. 023	. 066	. 125
17	. 077	. 077	000	125	- . 014	 001	. 105	. 125
18	109	110	001	- . 125	087	072	. 083	. 125
19	. 113	. 114	. 000	125	. 044	. 087	. 107	. 125
20	006	006	. 000	125	062	055	035	. 125

Table 2. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}=0.50$ (N=5, all angles normalized to $\theta_{\rm 3dB}$)

ACTUAL APPROX	DIFF		. SCATTE	RER LOC	ATION	
096 102	006	250	203	094	061	. 250
042 054	- . 011	250	. 005	. 075	. 083	. 250
-, 220 -, 20B	. 012	250	192	158	057	. 250
		250	. 105	. 111	. 176	. 250
		250	-, 242	214	. 071	. 250
		-, 250	056	-, 039	. 227	. 250
		-, 250	~. 212	. 052	. 199	. 250
		250	. 036		. 204	. 250
			189		. 142	. 250
		_		111	024	. 250
				. 062		. 250
						. 250
				. 101	. 124	. 250
					. 031	. 250
					_	. 250
		·				. 250
		·				. 250
						. 250
		- -			. –	. 250
						. 250
	096 102 042 054	096 102 006 042 054 011 220 208 . 012 . 122 . 126 . 004 940 -1. 774 835 459 445 . 014 172 171 . 001 . 116 . 110 006 263 277 014 . 057 . 053 . 006 . 047 . 044 003 . 141 . 113 028 . 088 . 087 001 . 031 . 036 . 004 666 400 . 266 . 381 . 354 017 305 284 . 021 . 032 . 040 . 008 . 075 . 076 . 001	096 102 006 250 042 054 011 250 220 208 012 250 122 126 004 250 940 -1. 774 835 250 459 445 014 250 172 171 001 250 116 110 006 250 263 277 014 250 057 053 006 250 047 044 003 250 047 044 003 250 141 113 028 250 088 087 001 250 088 087 001 250 666 400 266 250 381 354 017 250 305 284 021 250 032 040 008 250 075 076 001 250	096 102 006 250 203 042 054 011 250 005 220 208 012 250 192 122 126 004 250 105 940 -1. 774 835 250 242 459 445 014 250 056 172 171 001 250 212 116 110 006 250 036 263 277 014 250 189 057 063 006 250 189 057 044 003 250 181 141 113 028 250 008 088 087 001 250 008 088 087 001 250 004 301 036 004 250 229 381 354 017 250 177 305 284 021 250 149 032 040 008 250 189	096 102 006 250 203 094 042 054 011 250 005 075 220 208 012 250 192 158 122 126 004 250 105 111 940 -1. 774 835 250 242 214 459 445 014 250 056 039 172 171 001 250 212 052 116 110 006 250 036 038 263 277 014 250 189 045 057 063 006 250 189 045 057 063 006 250 181 062 141 113 028 250 008 067 088 087 001 250 008 067 088 087 001 250 094 101 031 036 004 250 229 150 381 354 017 250 177 177 305 284 021 250 149 001 032 040 008 250 189 141	076 102 006

Table 3. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}$ = .75 (N=5, all angles normalized to $\theta_{\rm 3dB}$)

REP	ACTUAL	APPROX	DIFF		. SCATTE	RER LOC	ATION	
1	981	-1.775	794	375	103	. 10B	. 173	. 375
2	470	454	. 016	375	087	. 004	. 290	. 375
3	. 186	. 171	015	375	278	. 198	. 286	. 375
4	. 258	. 284	. 025	- . 375	319	. 1 99	. 312	. 375
5	. 317	. 331	. 013	− . 375	. 005	. 016	. 199	. 375
6	. 345	. 346	. 001	375	. 314	. 339	. 343	. 375
7	. 675	. 443	232	<i>−.</i> 375	134	. 188	. 228	. 375
8	. 154	. 181	. 027	- . 375	112	. 252	. 309	. 375
9	. 223	. 253	. 030	− . 375	031	. 032	. 226	. 375
10	168	180	013	- . 375	154	118	. 276	. 375
11	. 235	. 311	. 076	375	339	233	. 150	. 375
12	. 201	. 214	. 013	- . 375	- . 325	311	. 180	. 375
13	. 163	. 172	. 010	375	295	006	. 022	. 375
14	023	013	. 009	- . 375	348	236	096	. 375
15	 103	113	- . 010	375	. 092	. 146	. 315	. 375
16	. 435	. 438	. 003	 375	. 072	. 334	. 342	. 375
17	. 040	. 045	. 004	- . 375	197	122	. 018	. 375
18	1.826	8. 418	6. 592	 375	064	. 067	. 096	. 375
19	. 114	. 151	. 036	375	232	041	. 128	. 375
20	. 261	. 271	. 030	375	. 017	. 077	. 079	. 375

Table 4. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}=1.00$ (N=5, all angles normalized to $\theta_{\rm 3dB}$)

REP	ACTUAL	APPROX	DIFF	• • • • •	. SCATTE	ERER LOC	ATION	
1	- . 170	128	. 042	500	352	230	. 350	. 500
2	. 129	. 174	. 044	 500	194	122	. 005	. 500
3	. 288	. 336	. 048	 500	319	237	. 316	. 500
4	. 172	. 356	. 184	500	497	198	. 481	. 500
5	289	277	. 012	500	448	282	207	. 500
6	247	300	053	500	375	. 024	. 037	. 500
7	. 230	. 182	048	500	454	. 090	. 247	. 500
8	438	437	. 001	500	 434	424	091	. 500
9	114	252	138	500	096	05B		
10	1.003	1. 775	. 772	5 00	088	061	. 295	. 500
11	. 221	. 216	005	500			. 259	. 500
12	. 569	. 071	499	500 500	380 - 360	. 057	. 112	. 500
13	018				29 9	. 154	. 227	. 500
_		054	. 015	 500	410	- . 150	. 438	. 500
14	. 352	. 221	130	500	221	052	. 492	. 500
15	. 149	. 078	- . 050	 5 00	 346	 313	. 180	. 500
16	- . 397	326	. 071	500	– . 377	. 295	. 455	. 500
17	 341	369	- . 028	- . 500	476	078	. 178	. 500
18	116	- . 097	. 020	- . 500	466	226	. 081	. 500
19	. 264	. 353	. 099	500	336	294	. 09B	. 500
20	. 378	. 416	. 018	500	161	152	309	500

Table 5. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB} = 1.25$ (N=5, all angles normalized to $\theta_{\rm 3dB}$)

REP	ACTUAL	APPROX	DIFF		. SCATTE	ERER LOC	CATION	
1 2 3 4 5 6 7 8 9	315 217	332	017 . 138 -3. 271 . 016 045 202 . 086	625 625 625 625 625 625 625 625 625	549 453 558 549 240 227 220 573 524 136	540 304 303 . 170 . 028 . 348 062 105 . 033 . 188	369 . 399 230 . 410 . 241 . 516 . 110 . 085 . 192 . 561	
11 12 13 14 15 16 17 18 19 20	. 496 . 222 429 051 . 005 . 077 -1. 712 400 . 010 260	. 241 131 1. 867 . 216 . 152 2. 483 279 453 . 018 384	255 352 2. 296 . 267 . 147 2. 406 1. 433 053 . 007 124	625 625 625 625 625 625 625 625 625	. 158 211 363 211 253 205 150 377 032 584	. 508 . 286 135 098 173 . 024 . 068 302 . 028 216	. 553 . 466 . 105 . 439 . 082 . 550 . 156 271 . 605 073	. 625 . 625 . 625 . 625 . 625 . 625 . 625 . 625

Table 6. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}$ = 1.50 (N=5, all angles normalized to $\theta_{\rm 3dB}$)

REP	ACTUAL	APPROX	DIFF	• • • • •	. SCATTE	RER LOC	ATION	
1	. 476	- . 155	631	 750	294	. 204	. 466	. 750
2	. 139	- . 236	- . 275	 750	634	299	. 029	. 750
3	091	14. 613	14.704	 750	542	200	. 463	. 750
4.	 093	2. 506	2. 598	750	51B	348	. 004	. 750
5	 138	- . 129	. 009	750	544	352	030	. 750
6	. 034	245	280	 750	637	264	. 575	. 750
7 .	064	. 343	. 407	750	296	. 097	. 380	. 750
8	-1. 985	274	1. 691	 750	510	. 558	. 618	. 750
9	. 090	026	117	 750	264	00 9	. 398	. 750 . 750
10	. 229	016	245	750	309	. 302	. 347	. 750
11	351	1. 839	2. 190	750	471	379	. 746	. 750
12	160	. 288	. 448	750	302	. 676	. 683	. 750
13	396	-2. 593	-2.198	750	 548	. 535	. 598	. 750
14	. 261	1.782	1. 521	750	641	01B	. 679	. 750
15	. 155	1. 781	1. 626	750	686	. 555	. 704	_
16			-9. 829	750	691	147	. 341	. 750
17	028	1.826	1. 854	750	266	232	. 749	. 750
18	. 023	221	244	750	743	295		. 750
19	005	. 433	. 438	750	708	016	. 678	. 750
20	352	301					. 748	. 750
20	372	301	. 051	- . 750	- . 479	. 386	. 740	. 750

Table 7. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}$ = .25 (N=10, all angles normalized to $\theta_{\rm 3dB}$)

REP ,	ACTUAL	APPROX	DIFF
1	. 014	. 014	. 001
2	. 033	. 033	. 000
3	029	029	. 000
4	. 042	. 042	000
5	. 032	. 031	001
6	- . 007	007	. 000
7	000	000	000
8	 014	CO6	. 007
9	. 010	. 010	- . 000
10	092	096	003
11	023	023	. 000
12	. 139	. 138	001
13	. 046	. 047	. 001
14	. 042	. 042	. 000
15	055	057	001
16	. 018	. 019	. 001
17	. 003	. 003	000
18	118	 117	. 000
19	 136	139	~. 003
20	056	058	002

Table 8. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}$ = .50 (N=10, all angles normalized to $\theta_{\rm 3dB}$)

			•
REP	ACTUAL	APPROX	DIFF
1	. 049	. 060	. 011
2	. 040	. 048	. 008
3	. 167	. 178	. 011
4	. 242	. 272	. 030
5	. 177	. 191	. 014
6	. 042	. 042	000
7	110	10B	
•			. 002
8	038	045	006
9	. 032	. 037	. 005
10	1. 030	1. 792	. 762
11	039	 049	- . 009
12	. 111	. 122	. 010
13	- . 156	 147	. 009
14	. 161	. 167	. 006
15	052	- . 055	002
16	206	218	012
17	. 028	. 027	001
18	 217	208	. 009
19	095	09B	- 004
20	121	103	
6 V	IÆI	103	. 018

Table 9. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}$ = .75 (N=10, all angles normalized to $\theta_{\rm 3dB}$)

REP	ACTUAL	APPROX	DIFF
REP 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16	ACTUAL 049 . 017 . 022 141 . 542 083 . 021 . 226 1. 156 . 188 043 . 282 . 055 . 119 . 192 232	APPROX 043 . 000 . 026 142 . 451 111 . 023 . 108 1. 825 . 224 043 . 356 . 053 . 093 . 228 303	006 016 . 004 001 092 028 . 002 118 . 669 . 036 . 000 . 074 001 026 . 036 072
16 17 18 19 20	232 . 587 . 247 . 036 309	303 . 036 . 271 . 031 279	072 551 . 043 005

Table 10. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}$ = 1.00 (N=10, all angles normalized to $\theta_{\rm 3dB}$)

REP	ACTUAL	APPROX	DIFF
1	045	096	051
2	. 048	. 068	. 020
3	013	. 003	. 016
4	. 681	2. 162	1. 481
5	426	442	- 016
6.	. 327	. 364	. 037
7	045	128	083
8	. 129	. 232	. 103
7	411	458	047
10	. 055	. 077	. 022
11	173	073	. 100
12	. 143	. 124	019
13	. 292	. 371	017 . 099
14	109	154	045
15	. 224	. 314	. 090
16	201	227	025
17	. 462	. 459	003
18	346	366	
19	. 203		020
20	036	. 414	. 211
	036	- . 021	. 015

Table 11. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_{\rm T}/\theta_{\rm 3dB}$ = 1.25 (N-10, all angles normalized to $\theta_{\rm 3dB}$)

REP '	ACTUAL	APPROX	DIFF
1	220	220	001
2	. 368	. 402	. 034
3	271	345	
4	. 077	. 138	. 061
5	. 187	258	445
6	184	455	- 271
7	. 027	1. 922	1. 895
ė	006	092	076
9			
	284	- . 356	072
10	- . 155	233	078
11	8 11	-2. 001	-1.190
12	229	277	048
13	. 248	. 216	032
14	. 041	025	065
15	363	431	069
16	070	177	107
17		-3. 174	-2. 499
18			
	136	. 441	. 577
19	058	−. 167	108
20	. 375	. 3 57	~. 018

Table 12. Comparison of Actual Target with Medium-Range Model (APPROX) for $\theta_T/\theta_{3dB} = 1.50$ (N=10, all angles normalized to θ_{3dB})

REP	ACTUAL	APPROX	DIFF
1	. 153	. 418	. 265
2	. 053	. 156	. 103
3	. 800	. 431	 369
4	. 242	. 456	. 214
5	165	. 170	
6	. 068		. 335
7		269	337
	222	439	217
8	. 155	. 452	. 297
9	. 048	−. 254	302
10	. 017	. 284	. 268
11	. 863	. 075	768
12	. 006	-2. 603	~2, 609
13	. 179	085	263
14	. 006	173	179
15	003	. 145	. 148
16	_	-1.774	-1.470
17	. 019	447	466
18	. 057	. 213	-
19	044		. 156
20		580	336
& V	. 140	. 204	. 064